

Problem 2

How many solutions to the PDE $u_t = u_{xx}$ can you find? Try solutions of the form $u(x, t) = e^{ax+bt}$.

Solution

There are an infinite number of solutions to this PDE. For example, there are polynomial solutions,

$$u(x, t) = 2t + x^2 \quad u(x, t) = 4t + 2x^2 \quad u(x, t) = 6t + 3x^2,$$

and there are exponential solutions of the form $u(x, t) = e^{ax+bt}$. Plug it into the PDE to determine a condition involving a and b .

$$\frac{\partial}{\partial t}(e^{ax+bt}) = \frac{\partial^2}{\partial x^2}(e^{ax+bt})$$

$$be^{ax+bt} = a^2e^{ax+bt}$$

Divide both sides by e^{ax+bt} .

$$b = a^2$$

Therefore, $u(x, t) = e^{ax+a^2t}$ is a solution of $u_t = u_{xx}$.